

M-ARY PHASE

SHIFT KEYING

M-ary PSK
Digital Modulation
Scheme.

Generalizes the concept
of Binary Phase Shift
Keying (BPSK) which
has 2 Phases $0/180^\circ$.

M-ary PSK $\Rightarrow$ M Phases.

M-ary PSK — M Possible Phases
Constellation has M symbols
 $\log_2 M$ bits/symbol.

M-ary PSK

$$0, \frac{2\pi}{M}, \frac{4\pi}{M}, \dots, \frac{2\pi(M-1)}{M}$$

M possible Phases.

$$i\text{th Phases} = \frac{2\pi i}{M}$$

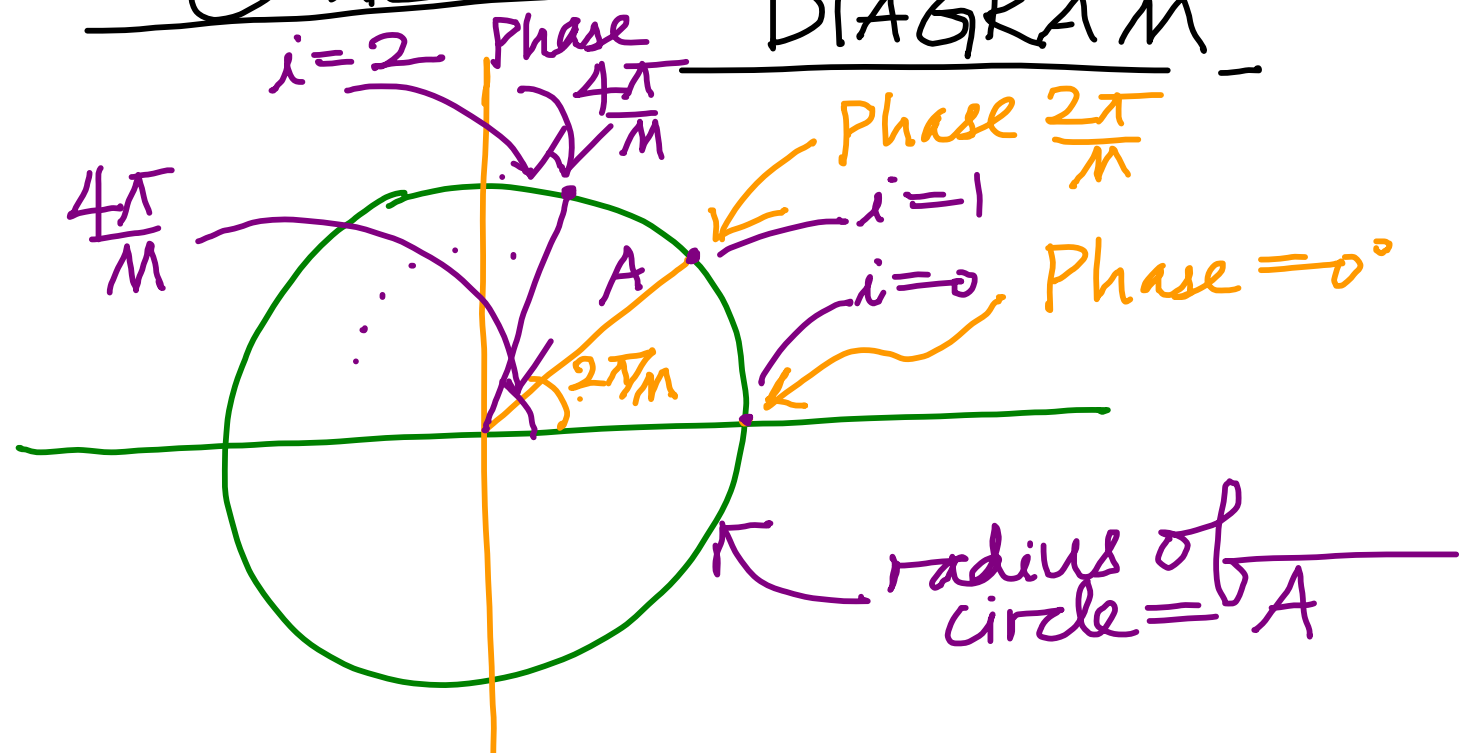
Dividing 2π into M Possible Phases. $0 \leq i \leq M-1$

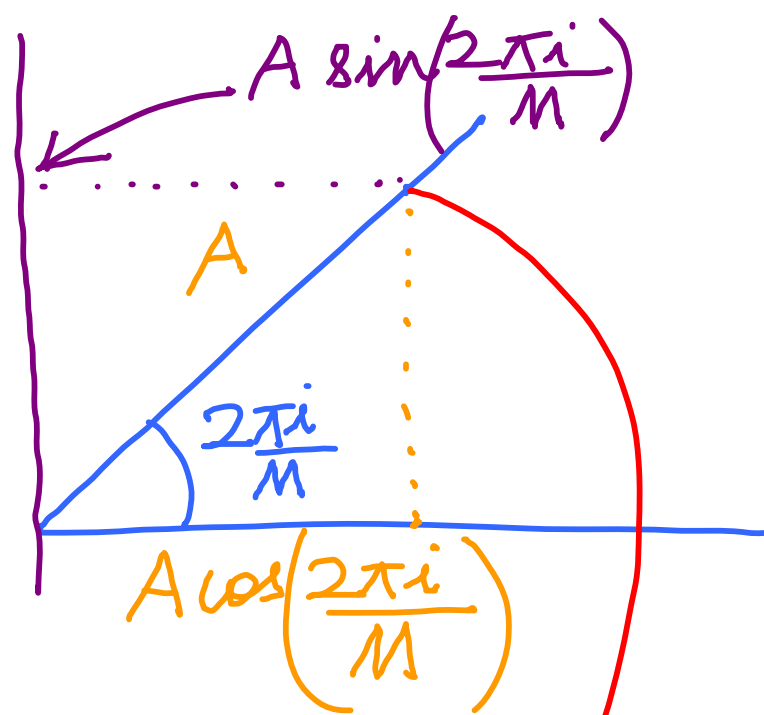
$$M=2, \quad 0, \quad \frac{2\pi}{2} = \pi$$

Phases are $0, \pi$
For $M=2$

Reduces to BPSK
For $M=2$

CONSTELLATION DIAGRAM





Constellation Pt i
 $a_1 = A \cos\left(\frac{2\pi i}{M}\right)$

Transmitted waveform $a_2 = A \sin\left(\frac{2\pi i}{M}\right)$.

$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$

$$a_1 \in \left\{ A \cos\left(\frac{2\pi k}{M}\right), k=0, 1, \dots, M-1 \right\}$$

$$a_2 \in \left\{ A \sin\left(\frac{2\pi k}{M}\right), k=0, 1, \dots, M-1 \right\}$$

$$\underline{p_1(t), p_2(t)}$$

orthonormal
basis Functions
for signal space.

$$p_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$p_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

$$0 \leq t \leq T$$

$$T = \frac{1}{f_c}$$

$$S_k = \left(A \cos\left(\frac{2\pi k}{M}\right), A \sin\left(\frac{2\pi k}{M}\right) \right)$$

$\nearrow$ k^{th} constellation pt

Average Symbol Energy

$$= E \left\{ \int_{-\infty}^{\infty} x^2(t) dt \right\}$$

$$= E \left\{ \int_{-\infty}^{\infty} (a_1 p_1(t) + a_2 p_2(t))^2 dt \right\}$$

$$= E \left\{ a_1^2 \underbrace{\int_{-\infty}^{\infty} p_1^2(t) dt}_{E_p} + a_2^2 \underbrace{\int_{-\infty}^{\infty} p_2^2(t) dt}_{E_p} \right\}$$

$$+ 2a_1a_2 \underbrace{\int_{-\infty}^{\infty} p_1(t)p_2(t)dt}_{0}$$

Inner Product
Since pulses are orthogonal.

$$= E \{ a_1^2 \mathcal{E}_p + a_2^2 \mathcal{E}_p \}$$

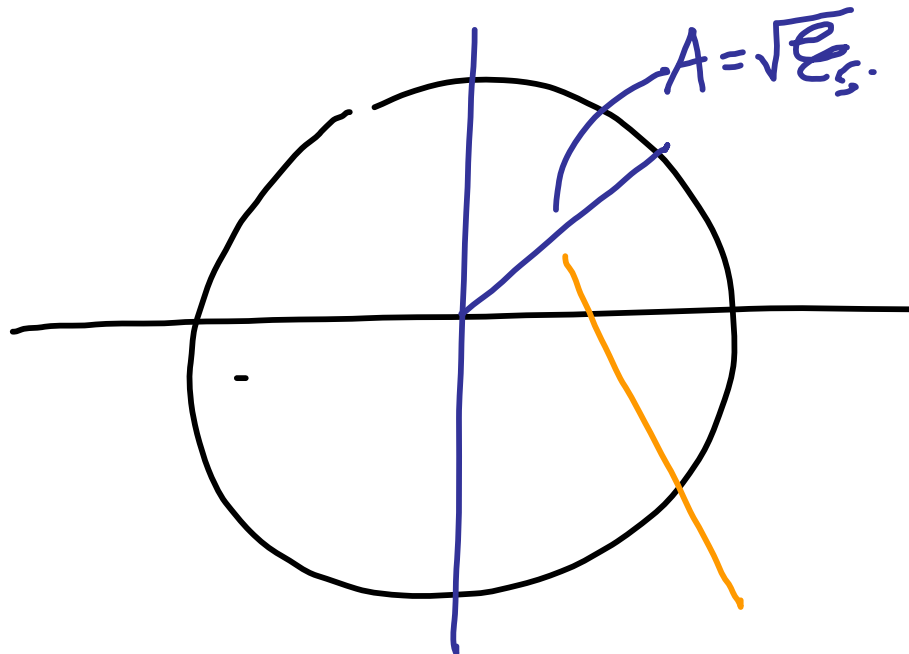
$$= E \{ a_1^2 + a_2^2 \} \quad \left\{ \begin{array}{l} \mathcal{E}_p = 1 \\ \text{unit Energy} \\ \text{pulse} \end{array} \right.$$

$$= E \left\{ \left(A \cos\left(\frac{2\pi k}{M}\right) \right)^2 + \left(A \sin\left(\frac{2\pi k}{M}\right) \right)^2 \right\}$$

$$= A^2 E \left\{ \cos^2 \left(\frac{2\pi k}{n} \right) + \sin^2 \left(\frac{2\pi k}{n} \right) \right\}$$

$$= A^2$$

$$E \{ x^2(t) \} = A^2$$



Setting symbol
Energy $= E_s$.

$$A^2 = E_s.$$

$$\Rightarrow A = \sqrt{E_s}.$$

radius of circle $= \sqrt{E_s}$.

✓ Each constellation pt
has energy = E_s .

$$S_k = \left(\underbrace{\sqrt{E_s} \cos\left(\frac{2\pi k}{M}\right)}_{a_1}, \underbrace{\sqrt{E_s} \sin\left(\frac{2\pi k}{M}\right)}_{a_2} \right)$$

Receiver Processing
For M-ary PSK

$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$
$$y(t) = x(t) + n(t) \quad \text{AWGN Gaussian Noise}$$